Hochschild-Mitchell (co)homology of a linear category, Galois coverings and skew categories

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Abstract

Let k be a commutative ring. A k-category $\mathcal{C}$ is a small category enriched over the k-modules. For Mitchell, $\mathcal{C}$ is a k-algebra with several objects. When a group G acts by k-autofunctors of $\mathcal{C}$, and if the action is free on the set of objects of $\mathcal{C}$, then the quotient category $\mathcal{C}/G$ exists (K. Bongartz, P. Gabriel, R. Martinez, J.A. de la Peña); $\mathcal{C}$ is then the Galois covering of $\mathcal{C}/G$. Otherwise the skew category $\mathcal{C}[G]$ always exists, and if the action is free on the objects, $\mathcal{C}[G]$ is k-equivalent to $\mathcal{C}/G$ (joint work with E. Marcos). This way the skew category can be considered as a substitute to the quotient category.

In a recent work with Eduardo Marcos, we compare invariants and coinvariants of the Hochschild-Mitchell (co)homology of $\mathcal{C}$ with certain direct summands of the ones of $\mathcal{C}/G$, in case the action on the objects is free. This comparison provides a frame and explains the existence of injective morphisms described in low degrees by E.L. Green, J.R. Hunton et N. Snashall. If the action is not free, we introduce an auxiliary category, with free action, which enable us to return to Galois coverings with the purpose of establishing analog results for the skew category.