Max-automorphisms of projections of exponent matrices

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Abstract

An integer $n \times n$-matrix $A = (\alpha_{pq})$ is called exponent if all its diagonal elements are equal zero and for all possible i, j and k the inequality

$$\alpha_{ij} + \alpha_{jk} \geqslant \alpha_{ik}$$

holds. The study of exponent matrices is important because they appear in Tiled orders theory.

Theorem 0.1. *[1] An arbitrary semi-maximal ring is isomorphic to a direct product of rings of the form*

$$\Lambda = \sum_{i,j=1}^{n} e_{ij}(\pi^{\alpha_{ij}}\mathcal{O}) \subseteq M_n(\mathcal{O}), \tag{1}$$

where $n \geqslant 1$, $\mathcal{O}$ is a discrete valuated ring with prime element π, (α_{ij}) is an exponent matrix, $e_{ij}(\pi^{\alpha_{ij}}\mathcal{O}) = \{e_{ij}(a), a \in \pi^{\alpha_{ij}}\mathcal{O}\}$ and $e_{ij}(a)$ is the $n \times n$-matrix whose unique non-zero entry a is placed in the (i, j)-position.

We pay attention to $n \times n$ non-negative exponent matrices $\mathcal{E}_n$. The set $\mathcal{E}_n$ is closed with respect to element wise maximum (called tropical sum $\oplus$) and element wise addition (called tropical multiplication $\odot$). Moreover the natural distributivity of $\oplus$ and $\odot$ holds.

We have obtained in [2] that any $\oplus$-automorphism and any $\odot$-automorphism of $\mathcal{E}_n$ is a composition of the transposing of matrix and the map $(\alpha_{ij}) \mapsto (\alpha_{\sigma(i)\sigma(j)})$ for some permutation $\sigma \in S_n$.

This work deals with co-projection of exponent matrices. For any subset M of indices of $n \times n$ matrix, let $\mathcal{E}_M$ be the set of all matrices from $\mathcal{E}_n$, which have 0 at all positions of M.

Theorem 0.2. *Any automorphism of $\mathcal{E}_M$, which preserves element wise maximum of matrices, is the restriction of a max-automorphism of $\mathcal{E}_n$.*

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