Interactions between graph theory and representation theory of algebras

José A. de la Peña
Instituto de Matemáticas
Universidad Nacional Autonoma de México
Ciudad de México, México
e-mail: jap@matem.unam.mx

Third International Colloquium: Alexander Zavadskij
Medellín, Colombia, June 28-29, 2018

Abstract

Given a (non-oriented) graph Δ, its it characteristic polynomial κ_Δ is defined as the characteristic polynomial of the adjacency matrix M_Δ of Δ. Observe that, since M_Δ is symmetric, all its eigenvalues are real numbers.

There are important interactions between the theory of graph spectra and the representation theory of algebras, due to the fact that if C is the Cartan matrix of $A = k[\vec{\Delta}]$, then M_Δ is determined by the symmetrization $C + C^t$ of C, since $M_\Delta = C + C^t - 2I$. We shall see that information on the spectra of M_Δ provides fundamental insights into the spectral analysis of the Coxeter matrix Φ_A and the structure of teh algebra A. For instance:

1. A fundamental fact for hereditary algebra $A = k[\vec{\Delta}]$, when $\vec{\Delta}$ is a *bipartite quiver*, that is, every vertex is a sink or source, is that $\mathrm{Spec}(\Phi_A) \subset \mathbb{S}^1 \cup \mathbb{R}^+$.

2. Simple constructions in graph theory provide tools to obtain in practice the characteristic polynomial of a graph. We recall two of them:

   (a) Assume that a is a vertex in the graph Δ with unique neighbor b and Δ' (resp. Δ'') is the full subgraph of Δ with vertices $\Delta_0/\{a\}$ (resp. $\Delta_0/\{a,b\}$), then

   $$\kappa_\Delta = x\kappa_{\Delta'} - \kappa_{\Delta''}.$$

   (b) Let Δ_i be the graph obtained by deleting the vertex i in Δ. Then the first derivative of κ_Δ is given by

   $$\kappa'_\Delta = \sum_i \kappa_{\Delta_i}.$$